

**Computational modelling dynamics of quantum and laser systems  
and backward-wave tubes with elements of a chaos**

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We numerically study nonlinear optics and dynamics of some quantum (atomic), laser systems and backward-wave tube in order to detect a chaos elements (quantum chaos). Many systems in a modern quantum physics and electronics manifest the elements of the deterministic chaos and hyperchaos in its dynamics. Chaos theory establishes that apparently complex irregular behaviour could be the outcome of a simple deterministic system with a few dominant nonlinear interdependent variables. Here we present the results of studying the dynamical chaos regime in generation of a laser with absorbing cell and chaotic self-oscillations in the backward-wave tube on the basis of numerical analysis by means a complex of advanced methods and algorithms (in versions [1,2]). In ref.[3] there have been presented the temporal dependences of the output signal amplitude, phase portraits, statistical quantifiers for a weak chaos arising via period-doubling cascade of self-modulation and for developed chaos at large values of the dimensionless length parameter. Our analysis techniques includes a multi-fractal approach, methods of correlation integral, false nearest neighbour, Lyapunov exponent's, surrogate data, memory matrix formalism [1,2]. In table 1 we present the data on the Lyapunov exponents' for two self-oscillations regimes in the backward-wave tube: i). the weak chaos (normalized length:  $L=4.24$ ); ii) developed chaos ( $L=6.1$ ). The correlations dimensions are respectively as 2.9 and 6.2. Our analysis confirms a conclusion about realization of the chaotic features in dynamics of the backward-wave tube. The same program is realized for detecting the chaos regime in generation of a laser with absorbing cell and multi-electron atoms in a microwave field.

Table 1. Numerical parameters of the chaotic self-oscillations in the backward-wave tube:  $\lambda_1-\lambda_6$  are the Lyapunov exponents in descending order,  $K$  is the Kolmogorov entropy (our calculation results)

Regime	$\lambda_1$	$\lambda_2$	$\lambda_3$	$\lambda_4$	$\lambda_5$	$\lambda_6$	$K$
Weak chaos $L=4.24$	0.261	0.0001	-0.0004	-0.528	-	-	0.261
Hyperchaos $L=6.1$	0.514	0.228	0.0000	-0.0002	-0.084	-0.396	0.742

- [1]. A.V. Glushkov, V.N. Khokhlov et al, Nonlin. Proc.in Geophys. 11, 285-294 (2004); Atm. Environment (Elsevier) 42, 7284–7292 (2008); Dynam. Systems – Theory. & Applications (Lodz, Poland) BIF-110 (2011); A.V. Glushkov, Methods of a chaos theory.-Odessa: Astroprint, (2012).  
 [2] V.D. Rusov, A.V. Glushkov et al, Adv. in Space Res. 42, 1614-1617 (2008); J. Atm. and Solar-Terr. Phys. (Elsevier) 72, 498-508 (2010).  
 [3] S.P.Kuznetsov, D.I. Trubetskov., Izv.Vuzov. Ser. Radiophys. XLVII, 1-7 (2004).